

The Specified Complexity of Retinal Imagery

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Abstract

An optical image is a very organized and specified collection of information governed by the laws of optics. The formation of an image, and its correct interpretation by sighted living creatures, is a unique example of the great complexity in the living world. While many other functional features of living organisms are extremely complex and point to the handiwork of a designing God, an optical image demonstrates a unique mapping process of the eye-brain system that is very useful to the organism. The transfer of light from an object scene to a visual detection system involving the eye and brain conveys an enormous amount of information. Unless that information is correctly organized into a useful image, however, the exchange of information is degraded and of questionable use. In this paper I examine the “connections” necessary for images to be interpreted correctly. I also address the additional complexity required for the dual-image mapping involved in stereovision. Statistics are presented for “simple eyes” consisting of a few pixels to illustrate the daunting task facing random-chance, purposeless, undirected evolution in the origin of any form of a functional eye. It is concluded that evolutionary processes cannot account for the perception of images by living organisms and that only a creator could produce complex visual systems.

Introduction

The object-image mapping process is quite complex and ubiquitous throughout nature in sighted creatures. Many organisms have the additional capability of overlapping fields of view providing for stereovision and depth perception. How imagery of one eye can be correctly sampled and reassembled to form a good image is

truly remarkable, while correlating two separate images, one from each eye, is astoundingly complex. It is worth examining the intricacy of this image-mapping process to determine if random processes could effectively account for the origin of vision.

The object-image mapping process seems to be dealt with sparsely in the literature, if at all. DeYoung (2002)

touched on this issue to some extent by describing an insect that has multiple eyes, each of which has a separate retina. The mapping process for that insect’s visual system first needs to invert each retinal image and then combine the various images into one contiguous field of view. For other *Creation Research Society Quarterly* papers dealing with eyes and vision, consult the references of Crofut and Seaman (1990), Hamilton (1985, 1987a, 1987b, 1988, 1991, 1993), and Sherwin and Armitage (2003).

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The Human Eye Compared to Images from Digital Cameras

The human eye (Hecht, 2002; Walker, 2000; and Smith, 1990) is roughly a 25-millimeter diameter sphere with a retina that contains about 120 million rods (black and white sensitive receptors) and about 6 million cones (color receptors). The region of greatest acuity is the foveola, which contains about 15,000 cones and is centered in the fovea. Today's digital cameras use a sensor made up of picture elements called "pixels," each of which detects light intensity, gray scale, and color. One could think of the structures of the retina in terms of pixels that sample the retinal image, as depicted in Figure 1. By "sample" I mean that the image falling on the retina gets divided into a large number of individual picture elements (pixels), all of which must be reassembled by the brain to reestablish a good image. Thus, the foveola could be thought of as a 125 by 125 pixel camera. At first glance this seems to be a very small

number of pixels compared to today's rather ordinary eight megapixel sensor cameras, which have something on the order of 3500 by 2300 pixels in their field of view (FOV). But the total "pixel" count for the human eye is about 126 megapixels, far beyond the 8-megapixel camera example. The cones of the fovea are individually connected to nerve fibers for high-resolution imagery. In this paper I will deal mostly with human eyes, but the basic premise will apply also to other sighted species with various forms of vision, such as compound eyes.

The Information Content of the Retinal Image

A couple of decades ago, images were analog for the most part, residing on a piece of film, or perhaps projected onto a screen. But with the advent of digital cameras, today's images are assembled by piecing together a large number of discrete pixels, each pixel making up only a small part of the overall image. When a digital image is finally assembled, the electronic and math-

ematical process involves a basic form of what is called "image processing" (Berry and Burnell, 2000), where each pixel's contribution to the final image can be changed or enhanced by many techniques. For example, the intensity of the output from a pixel can be increased, decreased, stretched, changed in color, etc. Noise (dust, scratches, low contrast) can be reduced by a number of mathematical techniques when all of the pixels are combined into a final image. But, no matter how the output from an individual pixel is changed, its precise location in the image itself must be preserved if no distortion (mapping error) is to be tolerated. Thus, image processing involves all of the details needed to first break an analog image apart into digitized components (pixels), perhaps then performing some image enhancement to the signal coming from each pixel, and finally reassembling the pixels in the right order to obtain a faithful representation of the object from which the image was made. In the human visual system, all of this image processing happens on a continuing basis over time. As the eye moves, the field of view changes, the lighting conditions vary, and the chemicals of the retina are continuously altered by the absorbed photons themselves.

A ballpark number for the image processing power of the human visual system can be estimated by the critical flicker frequency (CFF), which can be thought of as akin to the number of film frames projected per second by a movie theater projector. A standard TV updates the screen with 30 frames per second, for example. A retinal image digitized by ~100 million rods and cones responding at a modest CFF for a human eye of 10 pictures per second, results in a human image processing system that must deal effectively with ~ 10^9 responses per second. In this sense, the old adage is certainly true, that "an image is worth more than a

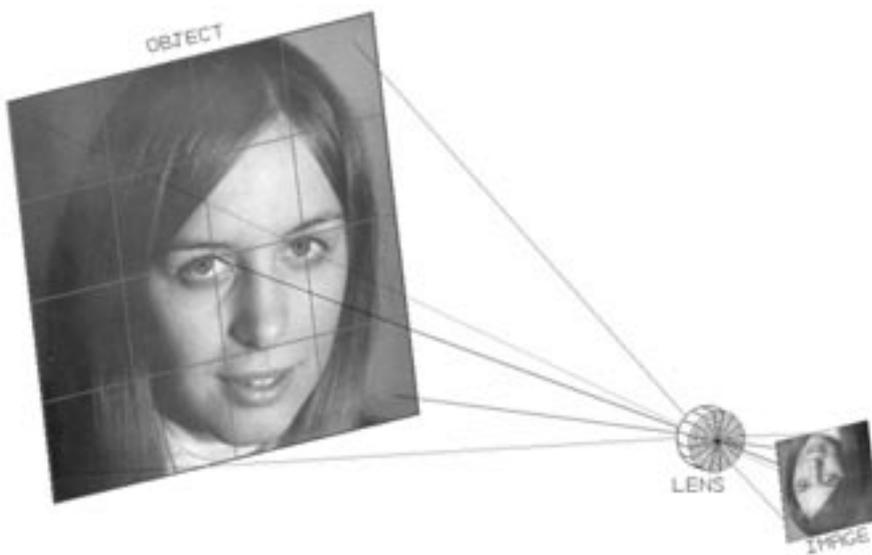


Figure 1. An object scene is viewed by a lens and an inverted image is formed in the focal plane of the lens. In a digital camera, a grid of light-sensitive elements (pixels) divides the image into a matrix. In the human retina, the matrix of rods and cones must be correctly connected to the brain in order for the image to be faithfully restored.

thousand words,” and an image from each eye (stereovision) even further compounds the image-processing task. The information transfer from an object scene to a sighted creature’s visual processing system is enormous and perhaps difficult to appreciate when simply calculating the previous approximate numbers.

Trying to Make Sense of the Image

The real problem for the human visual system is to “wire” the image proces-

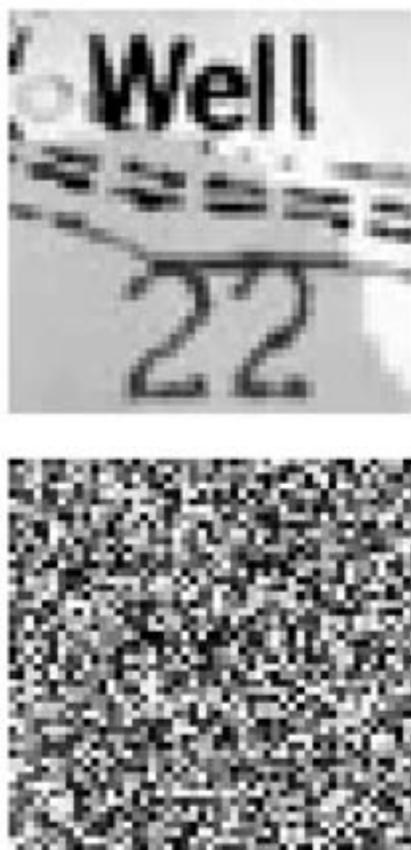


Figure 2. 50 by 50 pixel image of a small section of a topographic map. The correctly digitized (scanned) image is shown in (a), while the scrambled pixels shown in (b) represent one redistribution of the 2500! (1.63×10^{7411}) possible permutations of the pixels for this image. 50 by 50 pixels is roughly 1/6th of the foveal FOV for human eyes.

sor such that the visual information transfer is done correctly to yield a good image by which the perceived scene is a faithful representation of the object (Figure 2a). Figure 2b shows a scrambled version of the same pixels from Figure 2a, one of 2500 factorial ($2500! = 10^{7411}$) possible rearrangements or permutations of the 50 by 50 pixel array. An important question for an evolutionist to consider is how likely it is that the correct image arrangement of Figure 2a can be produced by “trial and error” processes in a visual system.

Examining the First Principles Only, with No Medical Details

In this paper I intend to provide examination of some first-order statistical numbers that help to constrain a visual system in terms of its complexity. The eye itself is not unlike a digital camera in that it samples an image through photoreceptors at a given frame rate, and is connected to an image processing system that attempts to make sense of what is being viewed. My use of terms such as “wired” or “pixel connections,” is merely for descriptive purposes as an actual organic vision system does not function in this precise fashion. A rigorous medical model is not being discussed here because it not only exceeds the purposes of this paper, but also because those additional biochemical and organic details only compound the problem of how an image is formed and has its content ultimately transferred to the brain for processing and interpretation. I will try to reduce a very complicated organic miracle to a much simpler engineering model, which will help demonstrate the complexity of sight based solely on correct image mapping. For the purposes of this paper, a rod or cone (pixel) gets connected to the brain (computer), and an image is nothing more than a collection of pixel connections that establish

a FOV. Clearly, a living organism’s visual system does not have pixels or direct wires that connect rods/cones to the brain. Many reference resources for the eye (Anonymous, 2005; Frisby, 1980) are filled with elaborate descriptions of the actual neural pathways and components that contribute to making an image; the reader is directed to these sources.

The evolutionary problem can be framed simply in terms of a digital camcorder hooked up to a television monitor. The camera (eye) views an object scene and sends the video information to the display (TV) in a specifically coded sequence such that the digitized images are ultimately organized and displayed as a “good image” on the monitor (the brain). This mapping process, from object scene to displayed image, could produce everything from a “perfect” image having complete correlation from object to image (Figure 2a), to a corrupted image such as the “snow” in Figure 2b, as well as anything in between. The details of the “wiring” of the camera to the processor are not important to the discussion here. I am evaluating only the final mapping of the output image compared to the input object scene in order to determine the quality of the image being presented by the complete system. This “mapping” of object scene to perceived image is the unique feature of a living visual system, and it is this mapping that will be the focus of discussion.

Some Statistics for Wiring the Eye by Random Chance

A 12-pixel Image

The number of possible combinations that an eye-brain system has in connecting all the rod/cone receptors is an astounding and virtually unknowable number. I shall start with a single wire

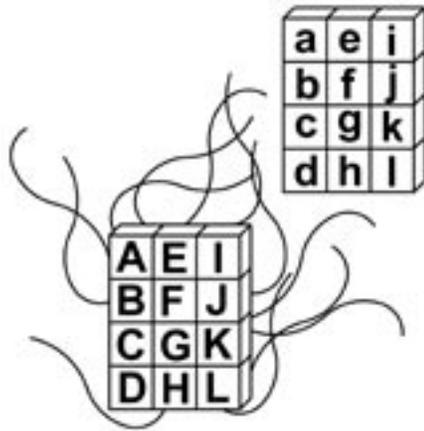


Figure 3. A 12-wire connection example where the same capital letter to lowercase letter is the correct connection, and all others have some level of error.

connection and then add additional “pixel” on the retina we wish to connect. Figure 3 shows an example of this kind of “eye-brain” wiring, for the case of 12 wires that need to be connected.

The correct connection of wires in the example shown is to have A-a, B-b, ...L-l pairings, such that an “image” that falls on the pixels in the capital letters block gets correctly transmitted to the lower-case block in exactly the right order. There are 12 factorial ($12! = 479,001,600$) permutations available in trying to make the connections, a task not easily accomplished by chance. Figure 4 shows how an “image” of the letter *F* would appear for the 12 pixels shown in Figure 3. If the connections are not done right,

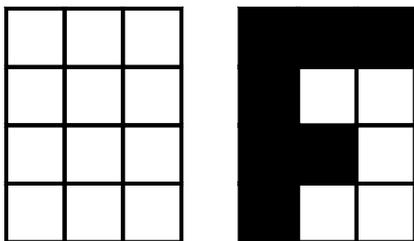


Figure 4. A letter ‘F’ image on the 12 pixels shown in Figure 3.

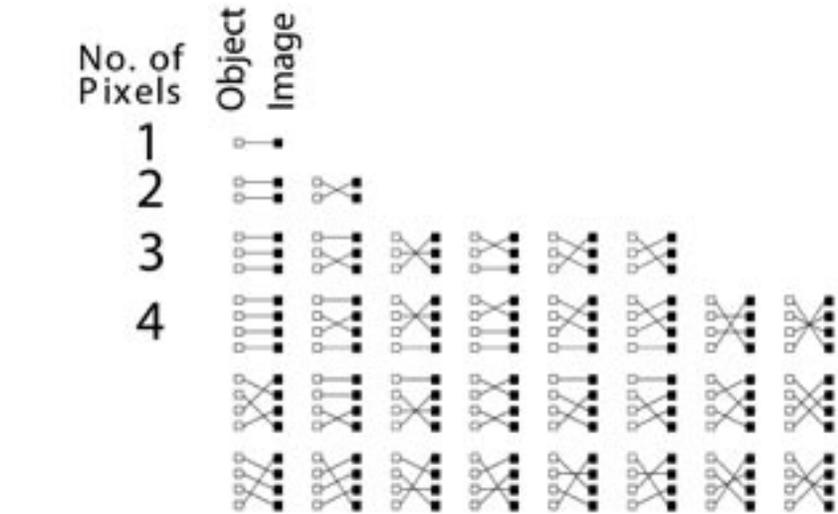


Figure 5. Wiring connection permutations for 1, 2, 3, and 4-pixel “eyes.”

however, what could be expected for the quality of the image in cases of incorrect wiring? There are almost 500 million permutations possible for the 12 pixels to be wired in different ways. To reduce the permutations to a manageable level, I shall start with some even simpler examples having fewer connections.

Some Statistics for Simple “Eyes” Having 1-4 Pixels

Figure 5 shows the possible connections for pixel counts ranging from one to four. For one pixel there is only one possible connection that can be made, so the accuracy of the image in this case is 100%. A one-pixel image is not very useful but is accurate in its connections.

For two pixels, there are two possible permutations: a correct wiring, and a completely incorrect one. The correct wiring has a 50:50 chance of occurrence. For three pixels, there are $3! = 6$ possible permutations, in which only one is correct, three have one correct wire in place, and two are completely wrong. In the case of four pixels, there are $4! = 24$ possible permutations. Only one case is completely

correct for all four wires; there are six cases where half the wires (two) are right, eight in which only one wire is right, and nine in which none are correct.

Examples of 1-11 Pixel “Eyes”

Figure 6 shows the results of cases for pixel counts ranging from 1 to 11. The trend is very clear: once the number of pixels starts to become more than just a few, almost all of the random wiring attempts are incorrect. Ignoring the left-most column of M, the first four columns tabulate the same number of permutations previously discussed. In the other seven columns of Figure 6, it becomes apparent that for larger numbers of pixels, the number of incorrectly connected pixels grows rapidly. At seven pixels and upward, very few correctly-wired pixels are added to the list in comparison to the huge number of additional incorrectly wired cases.

In Figure 7, the statistics for the case of 11 pixels have been listed and graphed to illustrate the trend of how the numbers are tracking (see Appendix 1 and 2). Where M is the number of correctly wired pixels, in the case of

The Convergence of Percentages for Correctly-Connected Pixels

It is somewhat surprising that the percentages tabulated in the second column of Figure 7 remain almost constant for increasing numbers of pixels in the FOV. These values obtain more significant digits of accuracy as the pixel count increases, but essentially the values given in Figure 7 represent the situation for any large number of pixels in the FOV. No matter how many pixels there are in the FOV, the percentage of correctly connected pixels remains fixed at $36.78794/M!$ percent. For a large number of pixels, the net evolutionary result would be possibly a few correctly connected pixels, but these would be lost within a huge ocean of incorrect connections. The visual system ultimately cannot tell which pixels are correctly wired and which are not, because the image tends to just look like noise. This intriguing aspect of the percentages converging to $36.78794/M!$ percent is shown graphically in Figure 8.

The remarkable aspect of the trend

shown in Figure 8 is that it is very improbable to obtain more than a handful of pixels correctly connected regardless of the number of pixels in the FOV, if chance evolution itself were responsible for having created the visual system. In the case of a 12-pixel FOV, for example, there are $12!$ possible permutations (479,001,600) of the pixel connections and only 1.5% of these will have four pixels correctly connected, as seen in Figure 8. In the case of the actual human eye fovea, which has 15,000 pixels, only 1.5% of those connection permutations would have four pixels correctly connected too, based on chance alone. Conversely, about 98.5% of the connection possibilities will be incorrect for the situation in which four pixels are hooked up right. So, for the case of the human fovea, how is the visual system supposed to be able to find those four correct wirings in the midst of the other 14,996 incorrectly wired receptors? If more than four correct connections are desired, the percentages depicted in Figure 8 drop by the

factor of $M!$, which exponentially limits the possibility of having a “workable” portion of the FOV provide a useful image. As an analogy, consider a town of 15,000 homes, only four of which have the correct addresses. It would be very difficult indeed to find the four correctly listed homes among the others. That is the same problem the brain would have in finding the correctly mapped rods and cones within a huge number of mis-wired receptors, if evolutionary mechanisms were used to first establish and then preserve the visual system.

Looking at More Complex Eyes

Almost all of the wiring combinations for an 11-pixel eye result in fewer than 6 pixels being correctly paired. Thus, 99.9% of all the combinations are incorrect wirings: a staggering concept for any chance-based process of image formation in a visual system. For the case of 11 total pixels, if roughly half the FOV needs to be wired correctly to arbitrarily make a “good” image, then a 6-pixel level shows that over 99.9% of the attempts to connect these six pixels correctly will be in error. Virtually all of the almost 40 million possible permutations for 11 pixels will be wrong, with less than half the FOV being wired correctly. In addition, if only one-fourth of the FOV needs to be wired correctly to constitute a “good” image, then the 3-pixel level of Figure 7 shows that over 93% of the possible permutations would still be wrong and would not give a good image in at least one-fourth of the FOV. So, even an 11-pixel FOV creates a situation in which the groups of 6 pixels or less having correct wiring represent 99.9% of the available combinations, and these percentages will not change as the number of pixels increases higher than 11. In the situation of 18 pixels in an “eye,” where six correctly wired pixels would represent one-third of the

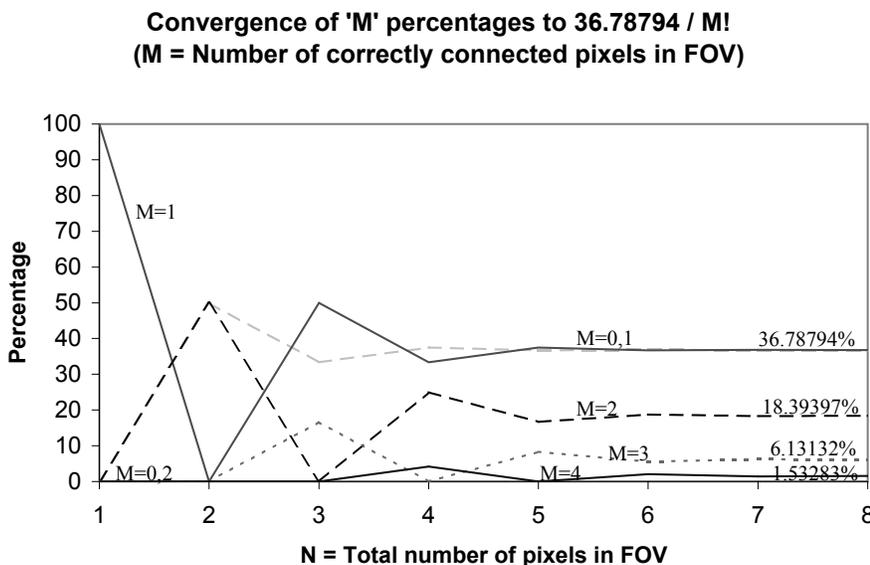


Figure 8. For 1-4 correctly connected pixels (M), the percentages of the pixel-connection permutations quickly become constant with increasing numbers of pixels in the FOV. The same trends apply to values of $M > 4$ where the percentages converge to $36.78794/M!$.

FOV, $18!$ permutations will have more than 6×10^{15} possible connections, and 99.9% of these will be wrong. So, more than two-thirds of the FOV will be incorrectly wired.

Extending this reasoning from 18 pixels to 60 pixels (less than an 8 by 8 pixel FOV) gives more possible permutations than the estimated number of particles in the universe (10^{80}), where $60! = 8.3 \times 10^{81}$. Again, 99.9% of all these possibilities will be the wrong combinations for six pixels out of 60 ($1/10^{\text{th}}$ of the FOV). Figures 6 and 7 show the difficulty of getting more than 6 correct connections out of 99.9% of the available possibilities by chance, and for any reasonable number of pixels in a visual system, the number of wrong connections quickly becomes staggering. The inverse of this huge number of wrong connections represents the “zero” probability of wiring a real eye by evolutionary processes. The same reasoning applies to any digital camera, so that no engineer would ever attempt to wire a digital imaging system by any random or “trial and error” process. What is the basis for the assumption that simply attempting “trial and error” over millions of years would eventually produce the human eye?

“Connecting” the Pixels of the Human Fovea to the Visual Processor (the Brain)

For the case of the high-resolution center of the fovea, where there are 15,000 pixels, the number of combinations ($15,000! = 2.75 \times 10^{56,129}$) is not a comprehensible number. Trying to correctly wire 15,000 pixels by a “trial and error” process is not realistically possible, and it would never be possible to get any part of the FOV of the eye to form an image that would be useful. Six correctly connected pixels leave over 99.9% of the $10^{56,129}$ combinations which are incorrect, and the odds are even worse for the chance

origin of more than six correctly wired pixels. Random wiring would produce an image not unlike the snow of a TV screen tuned to a nonfunctioning channel (see Figure 2b). Even if an evolutionary “trial and error” process were somehow able to get a few pixels correctly wired, the brain would have difficulty knowing that this condition had occurred because of the incredibly small portion of the FOV that is correctly wired. Natural selection would be unable to positively select for those few pixels in subsequent generations because they would effectively contribute nothing as yet to an image that can be discerned. Unless a substantial portion of the FOV of an eye is producing a useable image (i.e., a functional phenotype), there are no selectable features available for evolution’s process of natural selection. Without a selectable phenotype, it cannot distinguish between the correct and incorrect pixel wirings.

Examining a Real Human Eye and Two Eyes with Stereopsis

The problems associated with getting an image in the fovea to be useful are small compared to getting the entire FOV of an eye wired correctly. The immensity of a number such as $15,000!$ for the fovea becomes additionally incomprehensible when one tries mentally to wrestle with $126,000,000!$. Since evolution cannot work to achieve any long-range goal, it can only randomly pick combinations in a “trial and error” process. However, with only such a process available, how can evolution not only correctly wire one eye to the brain correctly, but also get a second eye to match exactly that specified pattern for the first eye? Even if one wants to argue about the definition of “exactly” in the matching process, Figures 6 and 7 show that one can accept a universe full of errors in

the matching of the two eyes and still not have a functional pairing of the two FOVs.

A second eye would contend with the same difficulties facing the origin of the first eye. Fusing two images into a combined stereo-pair places an additional burden of being mapped together. The process has already been depicted in Figure 3, but now the pairing of the capital to lowercase blocks represents a left eye to right eye mapping. As poor as the chances were previously for obtaining a useful image in one eye, the probability of two FOVs being mapped together correctly by any evolutionary process is even more troublesome.

There would not appear to be much of a tolerance for errors in these mapping processes either. For a single eye, the vernier acuity (Smith, 1990) of an observer is perhaps 5–10 times finer than the already high-quality foveal FOV resolution. Vernier acuity is the ability to align two objects, such as two straight lines next to each other in the FOV. Almost no “distortion” or image-mapping errors occur in this small central part of the retinal image, compared to what could be expected from randomly connecting the cones to the brain. When a normal observer uses both eyes for detailed observations, the mapping overlap of the two FOV’s appears to be continuous, again with no obvious and substantial mapping errors.

As another example of how random chance fails to produce complexity, Thompson (1990) presented the “monkeys typing all the books in the British Museum” statement of Huxley and concluded that this is nonsense. The interesting part of Thompson’s answer about reversing the question and asking how many words could actually be produced by the monkeys, leads to another remarkable find. If one replaces Huxley’s six monkeys with all the atoms in a 30 millimeter

ball bearing (lots of atoms) and each atom “types” at the speed of light for 20 billion years, there are less than 40 specified characters that result from all that effort. But the astounding part is that if we expand the problem to include the entire size of the supposed evolutionary universe (20 billion light-year radius, e.g.) and fill it with atomic computers, we get less than *three times* the number of characters produced by the 30 millimeter ball bearing. Adding more time and space to random-chance scenarios does virtually nothing to help the probabilities.

One just does not have the time or the population base in “evolutionary history” to produce any kind of complex functioning visual system based on random processes. Only a Creator could impart the information needed to provide meaningful sight, and that information would need to be available from the beginning, not acquired randomly by trial and error.

Other Considerations

Some readers could be concerned with my premise that a “unique wiring” is required as part of the specified complexity that defines the eye-brain visual system. Experiments have been conducted where patients have worn inverting eyeglasses, and after a lengthy period the brain was able to re-interpret the imagery as being right side up. But, just because the human visual system has the built-in complexity to be able to rewire itself does not negate the premise of this paper. Rather, it simply relocates the problem to the programming inherent in our DNA, wherein the visual system is capable of establishing or restoring imagery through a yet unknown process. This unknown process is absolutely not random because the visual system does not have enough time to search randomly through all the possible combinations. Rather,

that additional complexity has been built into the programming of the visual system such that it can overcome major obstacles with relative ease, despite the fact that we may never know exactly how it is done. Such rewiring experiments are forced to use FOVs that are simple transpositions of the original FOV, such as an image inversion. If, however, the FOV is allowed to be randomly scrambled, such as the situation shown in Figure 2b, the correct imagery would never be restored. It appears that God provided for the normal situation of an optical scene (or its inversion) on the retina to be mapped correctly with the code in our DNA, but did not design this process to be accounted for through random wiring connections. On the other hand, evolution requires random scenarios, and if evolution were true, it would seemingly be able to work with virtually any random-wiring assortment of the visual system. Since that certainly is not the case, the facts point once again to a Creator who established the process. So, how likely is it that the incredibly complex visual system of any sighted creature would develop from very simple eyes by chance-based evolution alone?

Some might point out that symmetry could exist in some of the cases shown in Figure 5, which could possibly change the statistics calculated in subsequent figures. In the case of two pixels, for example, the two possible ways of wiring the pixels either make a correct image, or an inverted one. The brain could probably work with either of these two scenarios and obtain a “good” image simply by inverting the one with upside-down wiring.

In the case of 9 pixels (3 by 3), the inverted-image symmetry case would exist, but there could be diagonal symmetry too. Thus, there could be rotational symmetry in the pixels, which might improve the chances for a good image to be perceived by the visual

system. For the case of the 9 pixels in a 3 by 3 matrix, it could be argued that there are actually 8 possible rotations of the image that could be useful, instead of the one case of “perfect” imagery. However, for 9 pixels, there are $9! = 362,880$ permutations of the image, and including the other cases of symmetry affects the statistical results only by 0.002%. While such cases of symmetry could be useful for a single eye, in the case of two eyes, all symmetry considerations are obviated. For stereovision, only a single orientation for the left-right images will work.

It could be argued that long ago a single-pixel eye and brain first worked to sense light and movement, and over the evolutionary eons all that mutations did was to add more and more pixels to the working eye. As shown earlier, for even a “trivial” 12-pixel eye there are almost 500 million possible ways to connect the pixels. In a world of “trial and error” processes, how do random mutations work to produce only the good combinations of connections and ultimately eliminate the bad ones? Even if chance could connect 3 of the 12 pixels correctly, in trying to get the fourth connection how does a mutation-driven process isolate the 3 correct pixels from subsequent variations? If evolution “mixes up” the wiring of what once was a well functioning eye into something less functional, what is the basis to suggest that the original functionality can be returned by random mutation alone?

Summary

The image-forming and image-processing capabilities of sighted living creatures are uniquely specified by the simplest laws of optics: the perceived image of an object scene must be a faithful mapping in order for the imagery to be useful and advantageous to the creature. When a retinal image is sampled and transmitted to the brain,

a unique “wiring” needs to be established not unlike that used for digital cameras. If this wiring process were performed randomly, even the simplest of sampled images would have an exponential number of possible connections from the actual image to its perceived or displayed counterpart. A surprising result from calculating the various permutations is that 99.9% of all the random attempts to correctly connect the first six pixels of the FOV are in error no matter how many pixels define the FOV. For a FOV the size of the human fovea (15,000 pixels), the conclusion is that 99.9% of the $10^{56,129}$ possible permutations for connecting this small part of the human FOV result in fewer than six pixels being correctly connected.

The mapping process for stereo-imagery from two eyes yields identical conclusions when the visual system attempts to correlate FOV's from each eye. The specificity of the neural network necessary for image formation/interpretation in any eye or pair of eyes cannot be explained by purposeless, undirected, evolutionary processes. Only a designing creator can create the visual systems capable of seeing and interpreting the information contained in the light that He created. Today's scientific research on visual systems shows that we are fearfully and wonderfully made.

Acknowledgement

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Appendix 1

The reader might be curious why an upper limit 11-pixel case is shown in Figures 6 and 7. To calculate the number of combinations that fell into each grouping M (0 to 11), the author wrote a computer program that searched all the possibilities one at a time. The almost 40 million combinations for 11 pixels (11!) took a 1GHz computer over three hours to calculate. Statistically, by 11 pixels, the trend in the data had sufficiently emerged and no higher values of N (>11) required the brute-force search.

Appendix 2

A recursion relationship exists for the values presented in Figure 6, as well as for additional values of N higher than N=11: $F(N, M) = (N/M) * F(N-1, M-1)$, where for the row of $M = 0$ the values are given by the Recontres Sequence: $F(N, 0) = (N!/e) + 0.5$, rounded to the next lowest integer, where $e = 2.71828...$ the base of the natural logarithms, and for the matrix locations $F(N, M=N) = 1$ and $F(N, M=N-1) = 0$. The accuracy of the values of $F(N, M)$ is only limited by the precision of the numbers involved.